Effect of Vortex Screening on the Bose Glass to Entangled Liquid Transition of Flux Lines in Superconductors

Ajay Nandgaonkar[1], D. G. Kanhere[1] and Nandini Trivedi[2]
[1] *Department of Physics, University of Pune, Pune 411007, India*
[2] *Department of Theoretical Physics, Tata Institute of Fundamental Research, Mumbai 400005, India*
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We study the phase diagram of flux lines in superconductors with columnar pins. Based on numerical exact diagonalisation simulations on small clusters, we get two phases of vortices: A low temperature pinned glass with diverging tilt modulus and a high temperature delocalised entangled vortex liquid. For random potential disorder we find a new phase transition temperature $T_{\rm BG}$ from a pinned Bose glass to an entangled liquid that reduces with increasing vortex density. This occurs primarily because vortices screen the disorder potential and generate an effective weaker random potential with increasing vortex density. For a fixed fraction of randomly placed attractive columnar pins of strength $V_{\rm pin}$, we find a Mott insulating phase when the vortex density exactly matches the number of pins ($B = B_\phi$). We also find a transition from a strongly pinned Bose glass for $B < B_\phi$ to a weakly pinned Bose glass for $B > B_\phi$ as the vortex density is varied.

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I. INTRODUCTION

The interest in equilibrium and dynamical properties of flux arrays in type II superconductors originates both from their technological importance and the rich variety of behavior these systems exhibit under different physical conditions. In a classic paper Abrikosov in 1957 [1] showed that for applied fields H such that $H_{c1} < H < H_{c2}$, the magnetic field penetrates in the form of quantized flux tubes ($\phi_0 = hc/2e$), which in the absence of disorder form a triangular lattice. In conventional low temperature superconductors, this Flux Line Lattice (FLL) was believed to exist at all temperatures upto $H_{c2}(T)$. With the discovery of high $\rm T_c$ superconductors, it was realized that, due to much higher transition temperatures, reduced effective dimensionality and short coherence lengths, fluctuations play an important role in deciding the structure and dynamics of FLLs [2]. One of the most significant consequences of enhanced thermal fluctuations on the FLL is its melting into a flux liquid phase via a first order transition [3,4].

Disorder arising from vacancies and interstitials, twin boundaries, grain boundaries and columnar pins also modify the structure and dynamics of the vortex lattice. The presence of strong static disorder is technologically relevant, for it leads to effective pinning of vortices thereby leading to high critical currents [2]. It also can lead to novel glassy phases such as the vortex glass and Bragg glass for the case of *random* quenched disorder [5–8].

Columnar defects i.e. linear damaged tracks in the material caused by heavy ion irradiation have emerged as very effective pinning centers [2,9]. In order to model the effect of columnar pins on the FLL, we exploit the mapping of 3D interacting flux lines onto bosons in (2+1)D [10,11]. In the mapped quantum problem, the columnar defects naturally map onto a time-independent random potential for bosons [12]. The irreversibility line can be interpreted as a phase transition where the vortex liquid at high temperatures freezes into a *Bose glass* (BG) of vortices pinned by columnar pins at lower temperatures. Path integral Monte Carlo simulations [14] find a low temperature BG with patches of ordered region with positional *and* orientational order which melts into an entangled defected liquid at high temperatures. Also, when the vortex density and defect densities are equal (at the matching field B_ϕ), each flux line is attached to one pin, leading to a Mott insulator (MI) phase [12]. Such a Mott insulator has been observed in magnetization relaxation experiments [15].

In this paper, we study, using numerical exact diagonalisation on small lattices, the different phases of flux lines with columnar pins for various densities of vortices and disorder strengths. We consider a lattice of N-sites with $N_v = H \times {\rm area}/\phi_0$ vortices, interacting via a hard core potential. We model disorder in two ways: (a) pinning disorder where a finite fraction of attractive pins, each of strength $V_{\rm pin}$, are placed randomly; and (b) a random disorder potential at each point. In case (a), we find that an entangled vortex liquid is stable against weak pins. For high pinning strengths, a Mott insulator is realised when the number of vortices is equal to the number of pins at the matching field B_ϕ. Signatures of a strongly pinned Bose glass and a weakly pinned Bose glass are also seen as the vortex density is tuned across B_ϕ. In case (b), we find two phases in the density-disorder plane. At low disorder an entangled vortex liquid which localizes into a pinned Bose glass with increasing disorder. We find that the critical disorder strength required to pin the vortex liquid increases with increasing vortex density. This implies that the temperature required to depin the vortices is reduced with increasing fields (see Fig. 7).

We organize the paper in the following way. In Section II we give the details of our model. In Section III A we discuss our results for pinning disorder, where we can

access different phases of vortices including the Mott insulator. In Section III B we discuss our simulations for the case where each site has a random disorder potential and conjecture an interesting experimental implication of our phase diagram.

II. THE MODEL

Consider a system of N_v flux lines in 3D in a magnetic field ($\mathbf{B}$) aligned with the z-axis, described by their 2D trajectories $\mathbf{r}_i(z)$ as they traverse a sample of thickness L with N_P columnar pins. Their free energy [12] is given by

$$\mathcal{F} = \int_0^L dz \sum_{i=1}^{N_v} \left\{ \frac{\tilde{\epsilon}_1}{2} \left| \frac{d\mathbf{r}_i(z)}{dz} \right|^2 + \frac{1}{2} \sum_{j \neq i}^{N_v} V[r_{ij}(z)] + \sum_{k=1}^{N_P} V_P[\mathbf{r}_i(z) - \rho_k^{\rm pin}] \right\}. \quad (1)$$

The first term in Eq. (1) is the line tension term with tilt modulus $\tilde{\epsilon}_1$. The second term denotes the interaction energy of all vortex pairs on a constant z-plane, where $r_{ij} = |\mathbf{r}_i - \mathbf{r}_j|$ and $V(r)$ the inter-vortex potential. The last term denotes N_P columnar pins ($\parallel \mathbf{B}$), modeled by z-independent potential V_P placed on randomly distributed positions $\{\rho_k^{\rm pin}\}$.

The classical statistical mechanics of Eq. (1) is equivalent to the quantum mechanics of interacting bosons interacting with a potential $V(r)$ in 2D with a random static potential $V_P(\mathbf{r})$. The partition function is determined by the ground-state energy of a fictitious quantum hamiltonian [11,12,16]. Using this mapping, the thermal fluctuations of the 3D-vortices get mapped onto the effective quantum fluctuations of bosons in two spatial dimensions and one imaginary time dimension. In this mapping, the temperature of the vortex system T plays the role of the Planck number $\hbar$. The bending energy $\tilde{\epsilon}_1$ of the flux lines is equivalent to the mass m of the bosons, so that like a lighter particle, a softer flux-line can wander more. The length of the vortices is equivalent to the inverse temperature *of* bosons so that in order to simulate a thick sample ($L \to \infty$) one considers the ground state of the quantum hamiltonian given by [12],

$$H = -t \sum_{ij} (a_i^\dagger a_j + h.c.) + \frac{1}{2} \sum_{i \neq j} V(r_{ij}) + \sum_i \mu_i n_i \quad (2)$$

where, operator a_i ($a_i^\dagger$) annihilates (creates) a boson at site i, $n_i = a_i^\dagger a_i$ is the number operator and μ is the chemical potential at site i. t is the measure of quantum fluctuations of bosons. μ_i has two parts: a uniform part $\mu \propto (H - H_{c1})$ which fixes the flux line density and $\delta\mu_i$ which represents site disorder.

The inter-vortex potential can be written as $V(r) = 2\epsilon_0 K_0(r/\lambda)$, with the modified Bessel function $K_0(x) \propto -\ln(x)$ as $x \to 0$, and $K_0(x) \propto x^{-1/2} \exp(-x)$ for $x \to \infty$. The energy scale is set by $\epsilon_0 = (\phi_0/4\pi\lambda)^2$, where $\phi_0 = hc/2e$ and λ the London penetration depth. In the low density limit i.e. when the inter-vortex separation (a_0) is large compared λ, the long range part of $V(r)$ dominates. In the absence of disorder, at low temperatures, the vortices form a triangular lattice. In the high density limit ($a_0 \leq \lambda$), the short ranged part of $V(r)$ decides the structure and elastic properties of the flux array. In the presence of columnar pins at low density, individual vortices get pinned whereas at high density, vortex bundles are pinned and the system breaks up into patches of ordered regions [14]. We use a simplified model and describe the inter boson interaction by a hard core interaction. In the low density limit, these bosons describe individual vortices whereas in the high density limit they describe the renormalized bundles of vortices.

We fix t to be unity and, disorder $\delta\mu_i$ is introduced in two ways: (a) *Pinning Disorder* : Attractive pins of strength $V_{\rm pin}$ are introduced at randomly chosen 25% of the sites, and (b) *Box Disorder* : Random disorder potentials at each lattice site chosen either from a uniform distribution of width $[-\Delta/2, \Delta/2]$ or from a gaussian distribution of width Δ. We work on a 4×4 square lattice with periodic boundary conditions, and use exact diagonalization techniques (such as Lanczos [17]) to compute the ground state properties of (2).

Our aim is to understand the combined effects of the strong intervortex hard core interaction and columnar disorder on the phases. The bosons described by Eq. (2) can exist in three different phases [13]: (a) Kinetic energy dominated superfluid phase for bosons that is equivalent to a high temperature entangled flux liquid in the vortex problem, where flux lines are delocalised and can hop freely from one columnar pin to another. (b) Disorder dominated Bose glass phase for bosons with a vanishing superfluid density ρ_s, that is equivalent to a Bose glass vortex phase with a diverging tilt modulus $c_{44} \propto \rho_s^{-1}$. (c) Interaction dominated gapped Mott insulator phase for bosons corresponds to the phase when the vortex density matches the density of columnar pins in the vortex problem. As opposed to a BG, the MI is incompressible since the vortex density remains locked to the density of pins over a finite range of external fields [12] and behaves like a Meissner Phase with $B = B_\phi$ instead of $B = 0$.

III. RESULTS

A. Pinning Disorder

On a 4×4 square lattice, we randomly choose a fixed fraction (say 25 %, $N_{\rm pin} = 4$) of sites to be pins, each of strength $-V_{\rm pin}$. In Fig. 1, we plot the superfluid density ρ_s/ρ, which is a measure of the stiffness of the phase of the order parameter, as a function of density ρ, for several values of $V_{\rm pin}/t$. Since we work on a lattice, $\rho_s/\rho \to 0$ as

$\rho \to 1$, and the repulsive local interactions give rise to a Mott insulator for any values $V_{\rm pin}$. In addition, for large $V_{\rm pin}/t$, the superfluid density goes to zero at a special Mott insulator point when $N_v = N_{\rm pin} = 4$; *the matching condition* where each columnar pin traps exactly one vortex. The distribution $P(n_i)$, of local density $\langle n_i \rangle$, follows a bimodal distribution, with a peak at 0 and another peak at 1. Note that the origin of this Mott insulator (for $B = B_\phi$) is different from the MI obtained in the jamming limit ($\rho \to 1$). This Mott insulator can be viewed as a Meissner phase for vortices at the matching field instead of zero field [12,18].

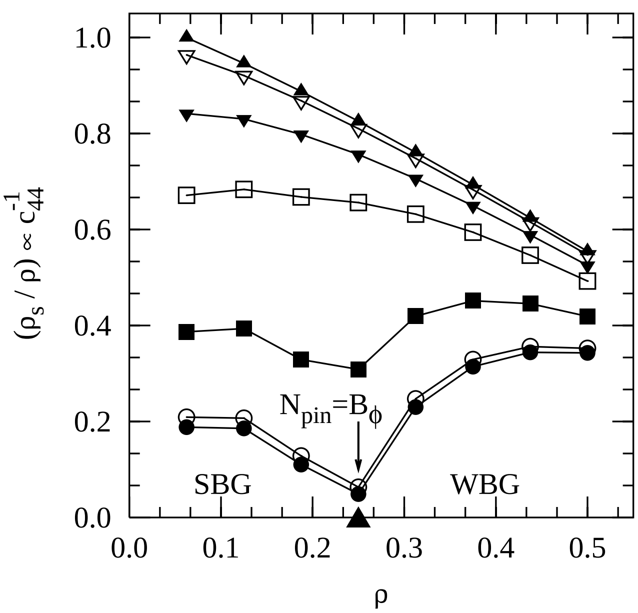

FIG. 1. **Pinning disorder**: ρ_s/ρ as a function of ρ for $V_{\rm pin} = 0.2, 1.0, 2.0, 3.0, 5.0, 7.5, 8.0$ (top to bottom). *Pins* of strength $-V_{\rm pin}$ are placed on randomly chosen sites. $N_{pin} = 4$. Data averaged over 300 disorder realizations. For high $V_{\rm pin}$ the Mott Insulator is realized at the matching field, when $\rho = N_{pin}$ (indicated by filled triangle on the x-axis).

For high $V_{\rm pin}$ (say $V_{\rm pin}/t = 8.0$ in Fig. 1), we note that

$$\frac{\rho_s}{\rho}\Big|_{B>B_\phi} > \frac{\rho_s}{\rho}\Big|_{B<B_\phi}, \tag{3}$$

implying the vortices are more entangled above the matching condition, than below. Alternatively, in Fig. 2 we plot binding energy per vortex defined as

$$B.E. = \frac{E(V_{\rm pin}) - E(0)}{N_v} \quad . \tag{4}$$

It is clear that above the matching field (B_ϕ) vortices are less bound by disorder than below. So depending upon the induced B, compared with B_ϕ, we find three phases within the BG, namely a strongly pinned Bose glass (SBG for $B < B_\phi$), a Mott insulator (for $B = B_\phi$), and a weakly pinned Bose glass (WBG for $B > B_\phi$), consistent with ref. [19].

In the SBG phase, vortices are collectively pinned by columnar pins [12,19], whereas in the WBG phase, one has patches of ordered regions [14]. It is interesting to see that calculations on such small systems, do exhibit signatures of these distinct glassy phases.

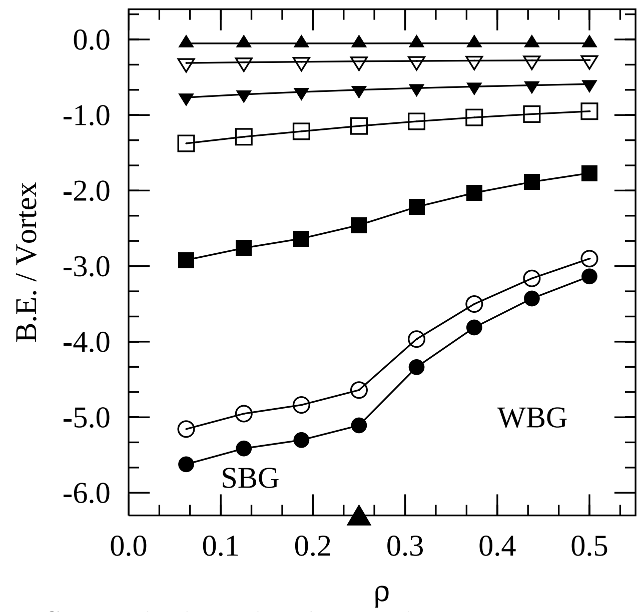

FIG. 2. **Pinning Disorder**: Binding energy per vortex as a function of density ρ for $N_{pin} = 4$. $V_{\rm pin}$ values same as in Fig 1. At large $V_{\rm pin}$, vortices are strongly pinned (the strongly pinned Bose glass phase) below the matching condition ($\rho = 0.25$) than above (the weakly pinned Bose glass phase).

B. Box Disorder

To further illustrate the effect of disorder on the FLL, we now present our results for the case where each site i on the lattice has a random potential Δ_i chosen from a uniform distribution of $[-\Delta/2, \Delta/2]$.

In Fig. 3 we plot the inverse tilt modulus $c_{44}^{-1} \propto \rho_s$ as a function of Δ for a fixed density of vortices ($\rho = 0.25$). For small Δ, c_{44} has a finite value (the entangled phase of vortices) and for large Δ it tends to diverge (the Bose glass phase of vortices). In the thermodynamic limit, it is expected that $\rho_s \sim |\Delta - \Delta_c|^\zeta$ with $\zeta = \nu z$ in 2D, where ν is the correlation length exponent, and z is the dynamical exponent [13]. However, in a finite system, the transition is rounded. As an indicator of the critical disorder strength Δ_c, we choose the point of maximum slope of $\rho_s(\Delta)/\rho$ (shown on the dotted curve at the bottom in Fig 3). A locus of Δ_c for different densities ρ gives us the phase boundary between the superfluid and the Bose glass (see Fig. 6).

For zero disorder, ρ_s/ρ differs from unity, which is a consequence of broken Galilean invariance. The kinetic

energy per particle along the direction of the twist was shown to be an upper bound to ρ_s/ρ [20]. We plot $\langle \mathrm{ke} \rangle_x/2t\rho$ as a function of Δ in the same figure. Near $\Delta = 0$ the kinetic energy per particle is exactly equal to the superfluid density and with increasing disorder it serves as an upper bound but being a local quantity, it fails to pick up the transition [20].

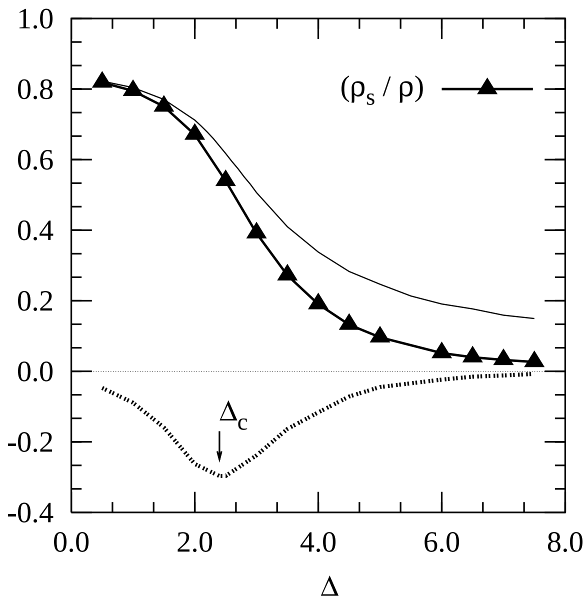

FIG. 3. **Box Disorder**: "Superfluid" density $\rho_s/\rho \propto c_{44}^{-1}$ (solid triangles) as a function of Δ for $N_v = 4; \rho = 0.25$. Slope of $\rho_s(\Delta)/\rho$ curve is shown at the bottom (dotted line). The point of maximum slope is identified as Δ_c (shown by an arrow). The kinetic energy along the direction of the twist, $\langle \mathrm{k.e.} \rangle_x/2t\rho$ (solid line), serves as an upper bound to ρ_s/ρ. Data averaged over 400 disorder realisations.

A quantity of interest to probe the spatial extent of the wavefunction is the inverse participation ratio defined as

$$I = \frac{\int |\psi_0|^4(r)dr}{\left(\int |\psi_0|^2(r)dr\right)^2} , \tag{5}$$

where, $\psi_0(r)$ is the ground state wavefunction of Eq. (2). For a localized state, I has a finite value, whereas for an extended state, $I \sim 0$. In Fig. 4 we plot I as a function of Δ for $\rho = 0.25$ and 0.50. $I \sim 0$ till Δ_c and starts rising above Δ_c indicating localization or pinning of vortices. For high disorder, the states are localized for all densities. Though it should be noted that, such a numerical scheme *cannot* really distinguish between an extended state and a localized state with large localization length (say for low Δ).

The degree of wandering of a flux line transverse to its length due to thermal fluctuations, can be obtained from the properties of the ground state wavefunction $\psi_0(r)$ in the mapped problem [12] by,

$$l_\perp^2 = \int d^2r \, r^2 \psi_0^2(r). \tag{6}$$

We plot the transverse wandering length per vortex in Fig. 5 for $N_v = 5$ and 8. Till Δ_c, $l_\perp$ is constant (almost equal to the clean case). For $\Delta > \Delta_c$, $l_\perp$ reduces, thereby marking localization of vortices due to disorder.

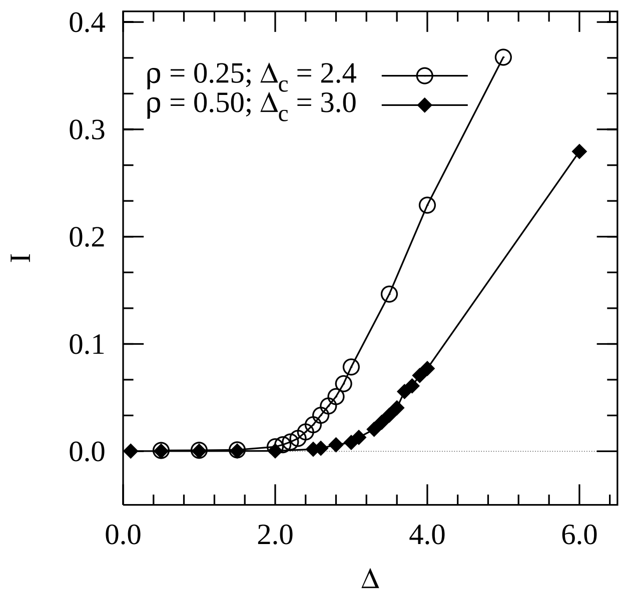

FIG. 4. **Box Disorder**: Inverse Participation Ratio I (see Eq.(5) for definition) which is a measure of the spatial extent of the boson wavefunction, for $N_v = 4$ and 8 as a function of Δ. $I \sim 0$ for entangled vortices ($\Delta < \Delta_c$) and finite for a pinned vortices ($\Delta > \Delta_c$).

The locus of critical disorder strength Δ_c, estimated from the behaviour of quantities discussed above as a function of Δ, for various densities (ρ) gives us the phase boundary between the superfluid (entangled liquid) and the Bose glass (pinned) in the density-disorder ($\rho - \Delta$) plane shown in Fig. 6.

Another measure of the critical disorder strength Δ_c (for a given ρ) is the disorder at which a finite fraction of *incoherent* sites occurs on the lattice. In a superfluid, phase coherence is achieved due to tunneling of bosons between different sites leading to fractional occupancies on sites. The long wavelength excitations are gapless, but due to the phase rigidity, local phase rotations cost a finite energy. We call a site *coherent* if the occupancy of that site is fractional. On the other hand, at high Δ, bosons are localised due to disorder. In this disorder dominated regime, local phase rotations do not cost any energy at a site which is integer occupied. We call a site *incoherent* if $\langle n_i \rangle = 0$ or 1, where $\langle n_i \rangle = 1$ corresponds to a single vortex pinned on the site i all along its length.

A locus of $\Delta_c(\rho)$ based on the emerging of an incoherent site concurs with the phase diagram shown in Fig. 6. The phase diagram, is re-entrant due to particle-hole symmetry about half filling ($\rho = 0.5$). At a mean

field level, it has been shown that the system becomes localized before clusters of nearly-incoherent sites percolate on the lattice [21].

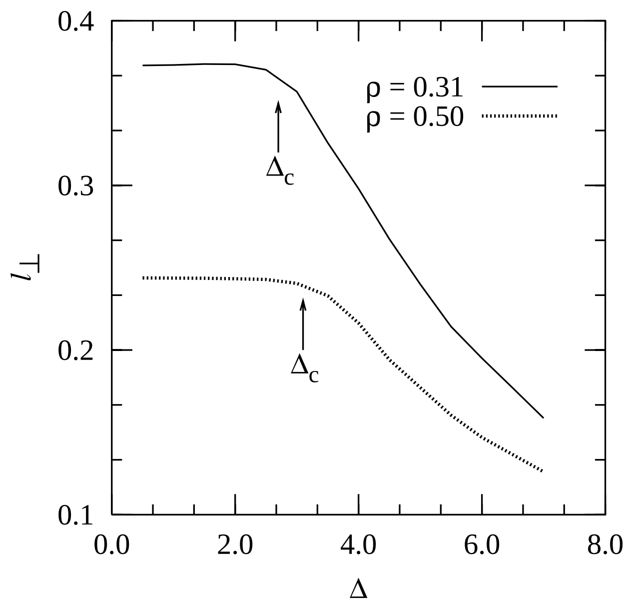

FIG. 5. **Box Disorder**: Transverse wandering length per vortex $l_\perp$ (see Eq. (6) for definition) as a function of Δ for $N_v = 5$ and 8. $l_\perp$ is a constant below $\Delta_c(\rho)$, almost equal to the clean case, and reduces above Δ_c marking pinning of vortices.

In this mean field theory, it is known that a Bose glass can never be accessed, because, a truly incoherent site with $\langle n_i \rangle = 0$ can be obtained only at $\Delta = \infty$ [22]. In ref. [20], by modeling the disordered boson problem as a network of random resistors, it was argued that in order to stop the flow of the current (or destroy superfluidity), a line of resistors perpendicular to the direction of the flow must be cut. In this perspective, we note that, our criterion acts as a *precursor* to the actual SF-BG transition. The true SF-BG transition would lie above our Δ_c and below the percolation transition (Δ_c^p) described in ref. [21].

The results presented above are for bounded disorder. For the case of unbounded (Gaussian) disorder, we find the same qualitative results as above. In some cases, Δ_c is marginally reduced relative to the Δ_c for the bounded disorder, since one can get large disorder values from the tails of the distribution.

Now we discuss an interesting experimental implication of our phase diagram. We see from Fig. 6 that for a higher density of vortices a higher disorder strength is required to prevent entanglement and wandering of the flux-lines. This happens because with increasing density (ρ), the disorder is effectively screened by vortices (strongest attractive sites are occupied first) and a new vortex introduced sees a weaker disorder landscape; thus is weakly pinned. Therefore, we conjecture that, *the transition temperature $T_{\rm BG}$, for a transition from a pinned Bose glass to an entangled liquid, will reduce with increasing vortex density.*

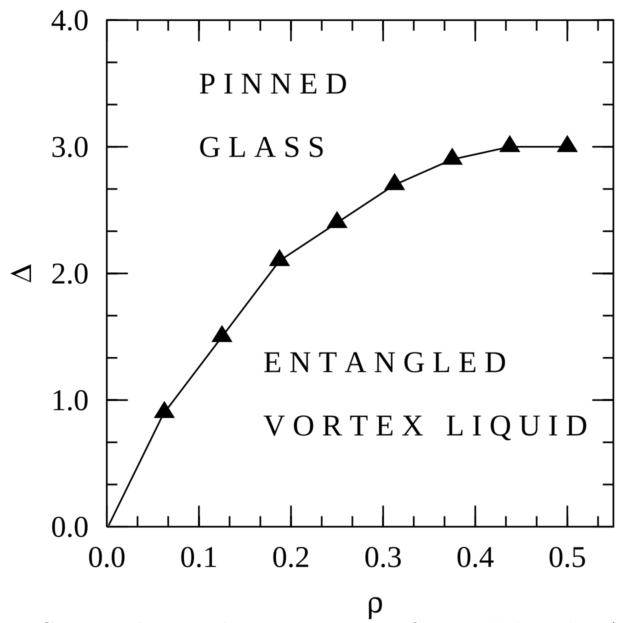

FIG. 6. **Phase Diagram**: Locus of critical disorder Δ_c, as a function of vortex density ρ. The line separates two phases: at low disorder, an entangled "superfluid" phase of flux lines and at high disorder a pinned "Bose-Glass" phase. The separatrix $\Delta_c(\rho)$ is determined by the emergence of a finite fraction of incoherent sites with $\langle n_i \rangle = 0$ or 1 Simulations are done on a 4×4 lattice, with periodic boundary conditions. Disorder ($\delta\mu_i$) is chosen from a uniform distribution of width $[-\Delta/2, \Delta/2]$. Results are averaged over 400 disorder realizations.

This conjecture is valid in two regimes in the $H - T$ plane. (a) At low fields, where the pinning energy dominates over interaction energy. For example, when the applied field is less than the matching field B_ϕ, columnar pins outnumber vortices and the pinning energy ($E_{\rm pin} \approx \epsilon_0 \ln(b/\xi)$, where b is the diameter of the columnar pin and ξ the coherence length) dominates over the magnetic repulsion [19]. We show this $T_{\rm BG}$ by a solid line in Fig. 7. The transition line we conjecture has a similar shape to the Nelson-Le Doussal line [24] *above* which point-defects play no role in deciding the structure of vortices. The physical reasons for the origin of Nelson-Le Doussal line and our $T_{\rm BG}$ are very different. Point defects "promote" flux line wandering and entanglement, whereas correlated disorder "inhibits" wandering and promotes localization. And it is precisely this role of columnar pins which leads to the new $T_{\rm BG}$ at low fields. (b) At high fields, where vortex bundle pinning dominates over individual vortex pinning [25]. In our model, since we neglect the long-range part in the intervortex interaction, we do not see signatures of vortex bundle pinning at high densities. For the case of vortex-

bundle pinning, the screening argument would still hold, and hence the topology of $T_{\rm BG}$ would be the same at high fields.

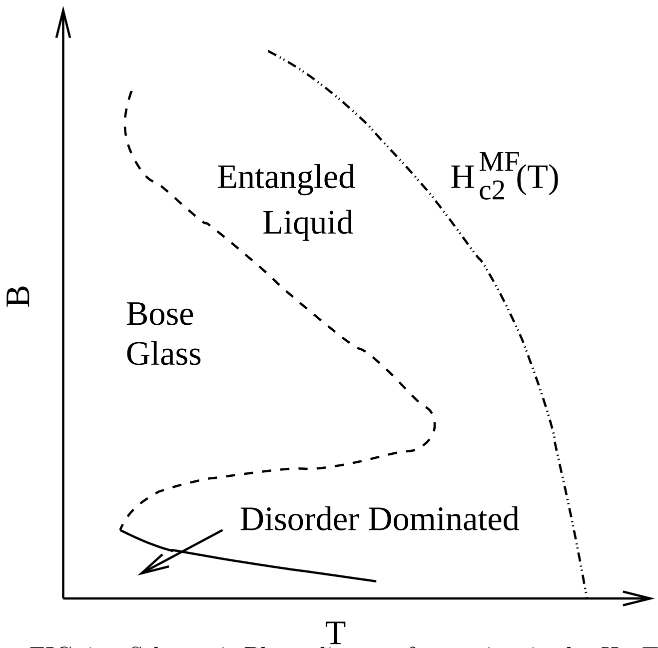

FIG. 7. Schematic Phase diagram for vortices in the $H-T$ plane. The flux line lattice melts much before $H_{c2}^{\rm MF}(T)$. The melting line $T_{\rm BG}$ separates a pinned Bose-Glass and an entangled vortex liquid. The dashed line is the re-entrant melting line. Our conjecture for the $T_{\rm BG}$ is shown by the solid line, in the disorder dominated, low H region. The figure is not drawn to scale.

At intermediate fields, the re-entrant "nose" (shown by the dashed line in Fig. 7) was predicted by Nelson [16] and has been experimentally verified [26]. We dont capture this re-entrant part in our simulations since we use an approximate form of the inter-vortex potential.

To conclude, we conjecture a double re-entrant phase diagram. Upon decreasing the field near the "nose" the vortex liquid localises into a glass where vortex bundles are pinned due to disorder and then delocalizes again into an entangled liquid. At lower fields, individual vortices are pinned by columnar defects, giving rise to another disorder dominated glassy phase.

Finally we note that, the bosonic Hubbard model is a paradigm for studying quantum phase transitions [27], in systems such as ^{4}He on random substrates, superconducting disordered thin films [28], magnets and Josephson-junction arrays [29]. The phase diagram in Fig. 6, for example is consistent with the field induced superconductor-insulator (SI) transition observed in coupled Josephson-junction arrays [29]. In low magnetic fields, vortices at $T = 0$ are pinned by disorder introduced via random offset charges. With increasing fields, the vortex density increases and at some critical density, vortices Bose-condense [30]. This vortex superfluid leads to an inifinite resistance, giving way to an insulator.

IV. ACKNOWLEDGMENTS

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